Offer for research internship 2026, 5-6 months
Sub-grid scale turbulence modeling for LES using the Lattice-Boltzmann method

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The Lattice Boltzmann Method (LBM) is a powerful tool for simulating fluid flows, capable of handling complex geometries thanks to Cartesian grids. LBM is a mesoscopic kinetic approach based on the discretization of particle velocities in space and time. Despite the low computational cost of LBM and the simplicity of Cartesian meshes, since the mesh nodes do not exactly follow the surfaces of solid boundaries, wall boundary treatment still poses a challenging topic. For high-Reynolds number flows turbulence modeling near the solid adds another layer of complexity to boundary treatment. This internship proposes the development and evaluation of different sub-grid scale turbulence models applied to an industrial LBM LES code. In particular, the adaptation of these models to the non body-conforming mesh near the wall will be the main focus of this work.

THE INTERNSHIP

In LBM we follow the evolution of the particle distribution functions f with different particle velocities $\boldsymbol{\xi}$, using the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \xi_\alpha \frac{\partial f}{\partial x_\alpha} = -\frac{1}{\tau}\left(f - f^{\mathrm{eq}}\right)$$

Which is an advection-relaxation equation, with f^{eq} being the Maxwell-Boltzmann distribution function at equilibrium, and τ being the relaxation time parameter. We can then recover the macroscopic quantities (density and momentum) from the moments of f in the velocity space:

$$\rho = \int f \ \mathrm{d}^3\xi, \quad \rho\mathbf{u} = \int \boldsymbol{\xi} f \ \mathrm{d}^3\xi$$

The internship will focus on the implementation of eddy-viscosity models to represent the sub-grid scale (sgs) turbulence. This class of models assumes that the effect of unresolved motions can be modeled as an additional effective viscosity, which enhances the dissipation of kinetic energy at the resolved scales. In LBM, this additional viscosity is taken into account by the relaxation parameter τ which is a function of the physical properties of the fluid, such as the viscosity:

$$\tau = 0.5 + (\nu + \nu_{sgs})/c_s^2$$

The internship will last for 5 to 6 months and will take place at the M2P2 laboratory (Laboratoire de Mécanique, Modélisation & Procédés Propres), UMR7340, Centrale Méditerranée, Plot 6, 38 rue Joliot-Curie 13451 in Marseille. Under the supervision of Pierre Boivin, Jérôme Jacob and Iason Tsetoglou, the intern will be responsible for the following tasks:

- Conduct a literature review on existing eddy-viscosity models and perform a critical assessment of their performance across various flow configurations.
- Implement selected models within the industrial Lattice Boltzmann code ProLB, with particular attention to the treatment of near-wall nodes.
- Compare and analyze results obtained from the different models through a series of validation test cases.
- Document the outcomes and develop a user guide summarizing best practices and recommendations on model selection for different flow scenarios.

THE CANDIDATE

The candidate should be a final-year student enrolled in an Engineering School (École d'Ingénieur) or Master's program (M2) with a specialization in fluid mechanics, aerospace engineering, numerical simulation, or applied mathematics. A solid foundation in fluid mechanics—and ideally in turbulence modeling—should be demonstrated through coursework, academic projects, or personal study. Programming skills are highly desirable (in C++, Fortran, MATLAB, or Python). The ideal candidate is rigorous, methodical, autonomous, and curious, with good command of English and a strong team spirit.

Your application, along with your CV and academic transcripts from the past two years, should be sent to the following addresses:

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BIBLIOGRAPHY

[1] T. Krüger, H. Kusumaatmaja, A. Kuzmin, O. Shardt, G. Silva, and E. M. Viggen, *The Lattice Boltzmann Method: Principles and Practice* (Springer International Publishing).

Offer of research internship 2026, 6 months
Extension of Variational-ALE Methodology to Multi-Fluid Flows

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The modelling of multi-phase and multi-fluid flows remains a key challenge in both academia (astrophysics, geophysics, meteorology, etc.) and industry (combustion, nuclear, health, etc.) due to the complexity of dispersed phases and their interactions. The "backbone" multi-fluid model, based on Euler-like equations for mass, momentum, and energy, provides a simplified framework that can be discretized numerically using the recently developed Geometry, Energy, and Entropy Compatible (GEEC) approach. This internship will be dedicated to extending the GEEC methodology to multi-fluid applications through systematic numerical validation on 1D and 2D academic test cases.

THE INTERNSHIP

This internship will focus on the extension and application of the GEEC numerical scheme to multi-fluid problems, with the incorporation of an Arbitrary Lagrangian–Eulerian (ALE) framework[1–4]. Starting from the discretized density, energy, and transport fields, the GEEC method ensures conservation of mass, momentum, and energy, preserves isentropic behaviour, and maintains thermodynamic consistency for each fluid. The internship will focus on implementing and validating these capabilities in a 2D parallelized solver for multi-fluid flows.

The internship will be structured around the following objectives:

- Study and implementation of numerical methods derived from a variational approach, particularly within the ALE framework.
- Proposal and execution of academic multi-fluid test cases in a 2D parallelized solver, with systematic comparison against benchmark solutions.
- Enhancement of the ALE framework to address stiff multi-fluid problems.
- Exploration and implementation of artificial viscosity strategies tailored for multi-fluid simulations.

The internship will be conducted within the M2P2 team, in collaboration with CEA researchers and engineers, combining theoretical, numerical, and practical aspects of multi-fluid simulations.

THE CANDIDATE

You are a final year École d'Ingénieur or Master 2 student specializing in fluid mechanics, numerical simulations, or applied mathematics. Knowledge of multi-phase flows is a plus. The candidate must have experience or interest in programming, particularly C++, and basic knowledge of parallelisation paradigms is welcome. Excellent English communication skills, both spoken and written, are required.

Your CV, cover letter, and grades for the last two years are to be sent to:
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BIBLIOGRAPHY

[1] T. Vazquez-Gonzalez, *Schémas numériques mimétiques et conservatifs pour la simulation d´´ecoulements* multiphasiques compressibles, Ph.d. thesis, Université Paris–Saclay (2016), ph.D. thesis 2016SACLC051.
[2] T. Vazquez-Gonzalez, A. Llor, and C. Fochesato, "A mimetic numerical scheme for multi-fluid flows with thermodynamic and geometric compatibility on an arbitrarily moving grid," European Journal of Mechanics B/Fluids **65**, 494–514 (2017).
[3] T. Vazquez-Gonzalez, A. Llor, and C. Fochesato, "A mimetic numerical scheme for multi-fluid flows with thermodynamic and geometric compatibility on an arbitrarily moving grid," International Journal of Multiphase Flow **132**, 103324 (2020).
[4] Heulhard de Montigny and A. Llor, "Taming the "stiff stiffness" of pressure work and equilibration in numerical schemes for compressible multi-fluid flows," International Journal of Multiphase Flow **153**, 104078 (2022).

Offer of research internship 2026, 6 months
Robust and accurate hybrid multiphase lattice Boltzmann model for incompressible two-phase flow

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The modeling of multiphase flow with the hybrid multiphase lattice Boltzmann model[1] is highly efficient; however, it usually suffers from numerical instability at large density ratios, Weber numbers, and Reynolds numbers. Our recent model[2] employs a consistent pressure scheme, high bulk viscosity, and a relatively thick interface width to stabilize the computation, but this approach simultaneously decreases accuracy. This internship will integrate advanced algorithms to enhance the numerical stability and accuracy of the hybrid multiphase lattice Boltzmann model.

THE INTERNSHIP

This internship will focus on developing a more robust and accurate hybrid multiphase lattice Boltzmann model for incompressible two-phase flow, where flow and phase fields are solved by the lattice Boltzmann method and the finite volume method, respectively, based on our recent recursive regularized multiphase lattice Boltzmann model. The lattice Boltzmann model for the flow field has been well established and will be further improved by the supervisor. The internship aims to incorporate a high-order finite volume solver into the existing numerical framework to accurately solve a phase-field equation, i.e., the Allen-Cahn equation

$$\begin{aligned} \partial_t C + \partial_\alpha \left(C u_\alpha\right) &= M \partial_\alpha \left(\partial_\alpha C - \Phi n_\alpha\right), \\ \Phi &= \frac{-4C\left(C-1\right)}{W}, \end{aligned} \tag{1}$$

where the volume fraction of the heavier fluid C is used as the order parameter, Φ is the potential well, W is the specified interface width, M is the mobility coefficient, n_α is the unit vector of the volume fraction gradient.

The internship will be structured around the following objectives:

- Study and implement a basic second-order finite volume solver for the Allen-Cahn equation, and validate the code by simulating some benchmark cases.
- Apply the high-order weighted essentially non-oscillatory (WENO) scheme[3] (both third-order and fifth-order) to discretize the convection term $\partial_\alpha \left(C u_\alpha\right)$.
- Determine the optimal discretization scheme for the diffusion and anti-diffusion terms on the right side of the Allen-Cahn equation, and the general principle of selecting the mobility coefficient.
- Exploration and implementation of the hybrid multiphase lattice Boltzmann model for a simplified engineering case, e.g., 3D jet flow[4].

The internship combines theoretical, numerical, and practical aspects of multiphase flow simulations. It will be carried out at the M2P2 laboratory, as part of the project of the ANR Liberty Industrial Chair.

THE CANDIDATE

You are a final-year student in an École d'Ingénieur or enrolled in a Master 2 program, specializing in fluid mechanics, numerical simulation, or applied mathematics. Basic knowledge of multiphase flows and the finite volume method is required. Experience or a strong interest in programming (Fortran or C++) is an asset, and knowledge of parallel computing paradigms is welcome. Good English communication skills, both spoken and written, are required.

Your CV, cover letter, and grades for the last two years are to be sent to:
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BIBLIOGRAPHY

[1] T. Gregorczyk, S. Zhao, and P. Boivin, "Hybrid lattice boltzmann method for multiphase flows," Physical Review E **112**, 025311 (2025).
[2] J. Lu, T. Gregorczyk, S. Zhao, and P. Boivin, "Phase-field-based recursive regularized multiphase lattice boltzmann model with a consistent pressure scheme," International Journal of Multiphase Flow , 105500 (2025).
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[4] S. Saito, A. De Rosis, A. Festuccia, A. Kaneko, Y. Abe, and K. Koyama, "Color-gradient lattice boltzmann model with nonorthogonal central moments: Hydrodynamic melt-jet breakup simulations," Physical review E **98**, 013305 (2018).

Offer of research internship 2026, 6 months
Post flame CO-oxidation modelling in large-scale industrial burners using the Lattice-Boltzmann method.

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In our recent simulations of a semi-industrial-scale burner using the Lattice-Boltzmann method, the reduced chemistry comprised of one-step schemes for methane and hydrogen, with NOx estimated a posteriori using the extended Zeldovich mechanism. Because these schemes produce CO but omit post-flame CO $\rightarrow$ CO_2 oxidation pathways, CO is systematically over-predicted, affecting temperature and radiative heat-transfer fields. Therefore, this internship will be dedicated to the development of a computationally affordable treatment of post-flame CO oxidation for CH_4/H_2 premixed combustion within an LBM–LES framework, and demonstrate its effect on predicted CO levels, temperature, wall heat flux, and NOx assessment.

THE INTERNSHIP

Rapid decarbonisation and pollution reduction requires designing industrial burners that run on lower-carbon fuels - such as H_2-enriched natural gas - while preserving safety, operability, and performance. To this extent, a semi-industrial burner designed to operate on hydrogen-enriched natural gas (CH_4/H_2 blends) has been developed by Fives to provide a pragmatic pathway for near-term decarbonisation of industrial burners while remaining compatible with existing plant constraints. However, this leads to altered flame stabilisation, post-flame oxidation, and radiative heat-transfer characteristics.

Under the Liberty project, the Lattice-Boltzmann method (LBM)[1] is employed to simulate this geometry, owing to its computational efficiency on complex industrial meshes and its compatibility with LES-based combustion closures. Within this framework, a gray P_1 radiation model has been implemented and applied to the Fives burner (6 m), establishing a baseline for radiation-flow-chemistry coupling in a confined, high-temperature configuration. In this study, a reduced chemistry comprising one-step schemes for methane and hydrogen, with NO_x estimated a posteriori via the extended Zeldovich mechanism, was used. As a result, post-flame CO $\rightarrow$ CO_2 oxidation[2] was not represented, leading to systematic CO over-prediction, and potential discrepancies in the recovered temperature field and wall heat-flux distribution.

The proposed internship is situated within the broader effort to deliver predictive, cost-effective modelling tools for hydrogen-enriched industrial combustion. It will address the identified gap by developing and integrating a computationally affordable treatment of post-flame CO oxidation compatible with LBM-LES of CH_4/H_2 burners, validating it against canonical and semi-industrial cases, and quantifying its implications for CO levels, temperature, wall heat flux, and downstream NO_x assessment. The results will also be used to review the limitations of the gray P_1 closure and to propose higher-fidelity radiation models as the basis for a subsequent PhD.

This 5-6 month research internship will take place in 2026 at Laboratoire de Mécanique, Modélisation & Procédés Propres (M2P2), UMR7340, Centrale Méditerranée, Plot 6, 38 rue Joliot-Curie 13451 in Marseille, under the supervision of Pierre Boivin and Song Zhao, and in cooperation with Uday Chikkabikkodu.

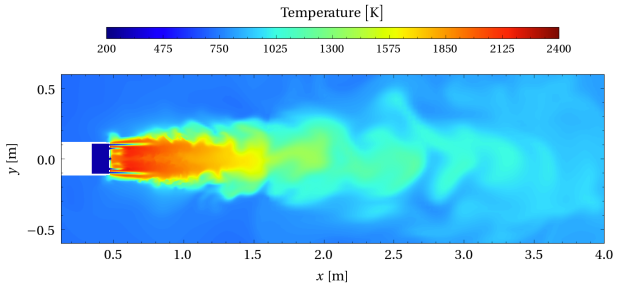

FIG. 1 Instantaneous temperature field.

THE CANDIDATE

You are a final year École d'Ingénieur or Master 2 student specializing in fluid mechanics, numerical simulations, or energetics. The candidate must have experience or interest in programming, particularly C++. Excellent English communication skills, both spoken and written, are required.

Your CV and grades for the last two years are to be sent to:
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BIBLIOGRAPHY

[1] M. Tayyab, *Development of Combustion Modelling within Lattice Boltzmann Framework.*, Ph.d. thesis, Aix-Marseille University (2020).
[2] B. Adams, M. Cremer, and D. Wang, "Modeling non-equilibrium co oxidation in combustion systems.".